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CRITERIA FOR THE REJECTION OF OUTLYING OBSERVATIONS

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Indian Standard

CRITERIA FOR THE REJECTION OF OUTLYING OBSERVATIONS

0. FOREWORD

0.1 This Indian Standard was adopted by the Indian Standards Institution on 25 July 1978, after the draft finalized by the Quality Control and Industrial Statistics Sectional Committee had been approved by the Executive Committee.

0.2 An outlying observation or an 'outlier' is one that appears to deviate markedly from the other observations of the sample in which it occurs. An outlier may arise merely because of an extreme manifestation of the random variability inherent in the data or because of the non-random errors, such as gross deviation from the prescribed experimental procedure, mistakes in calculations, errors in recording numerical values, other human errors, loss of the calibration of an instrument, change of measuring instruments, etc. If it is known that a mistake has occurred, the outlying observation must be rejected irrespective of its magnitude. If, however, only a suspicion exists, it may be desirable to determine whether such an observation may be rejected or whether it may be accepted as part of the normal variation expected.

0.3 The procedure consists in testing the statistical significance of the outlier(s). A null hypothesis (assumption) is made that all the observations including the suspect observations come from the same population (or lot) as the other observations in the sample. A statistical test is then applied to determine whether this null hypothesis can be rejected at the specified level of significance (*see* **2.8**). If so, the outliers can then be taken to have come from a population(s) different from that of the other observations in the sample and hence the outlier(s) can be rejected.

0.3.1 This standard provides certain statistical criteria which would be useful for the identification of the outlier(s) on an objective basis. When so identified, necessary investigations may also be initiated wherever possible to find out the assignable causes which gave rise to the outlier(s). The other objectives of the statistical tests for locating the outlier(s) may be to:

a) screen the data before statistical analysis;
b) sound an alarm that outliers are present, thereby indicating the need for a closer study of the data generating process; and
c) pin-point the observations which may be of special interest just because they are extremes.

0.4 When a test method recommends more than one determination for reporting the average value of the characteristic under consideration, the precision (repeatability and reproducibility) of the test method is found to be useful in detecting observations which deviate unduly from the rest. For further guidance in this regard, reference may be made to IS : 5420 (Part I)-1969*. When the precision of the test method is not known quantitatively, one or more of the procedures covered in the standard may be found helpful. The tests outlined in this standard primarily apply to the observations in a single random sample or the experimental data as given by the replicate measures of some property of a given material.

0.5 Although a number of statistical tests based on various considerations are available to screen the given data for outliers, only those tests which are simple and efficient have been included in this standard.

0.5.1 In the case of a single outlier (the smallest or the largest suspect observation), two tests are available. One test is based on the standard deviation whereas the other test is based on the ratio of differences between certain order statistics, that is, the observations when they are arranged in the ascending or descending order of magnitude. The latter test would be more useful in those cases where the calculation of standard deviation is to be avoided or a quick judgement is called for.

0.5.2 In the case of two or more outliers at either end of the ordered sample observations, the test given is based on the ratio of the sample sum of squares when the doubtful observations are omitted to the sample sum of squares when the doubtful observations are included, the sample sum of squares being defined as the sum of the squares of the deviations of the observations from the corresponding mean. If simplicity in calculation is the prime requirement then the test based on the order statistics for the case of single outliers may be used by actually omitting one suspect observation in the sample at a time. However, this test is to be applied with caution because the overall level of significance of the test may change due to its repeated applications.

0.5.3 In the case of two or more outliers such that one is at least at each end of the ordered sample observations, two tests have been given. One test is based on the ratio of the range to the standard deviation whereas the other test is based on the largest residuals, a residual being defined as the deviation of an observation from the corresponding mean. It may be mentioned that the former test is applicable when only two observations, namely, the largest and the smallest observations, are suspect whereas the latter test is much more general.

0.6 In a given set of data, whenever a large number of observations (say more than 25 percent) are found to be outlying, it may be desirable to

*Guide on precision of test methods: Part I Principles and applications.

discard the entire data. However, this guideline is to be applied with considerable caution in those situations where the data consists of a few observations only.

0.7 Almost all the statistical criteria for the identification of the outliers as given in this standard are based on the assumption that the underlying distribution of the observations is normal. Hence, it is important that these criteria are not used indiscriminately. In case the assumption of normality is in doubt, it is advisable to obtain the guidance of a competent statistician to ascertain the feasibility of the applicability of these test criteria.

0.8 In reporting the result of a test or analysis, if the final value, observed or calculated, is to be rounded off, it shall be done in accordance with IS : 2-1960*.

1. SCOPE

1.1 This standard lays down the criteria for the detection of outlying observations in the following three situations:

a) Single outlier (at either end of the ordered sample observations),
b) Two or more outliers (at either end of the ordered sample observations), and
c) Two or more outliers (one at least at each of the two ends of the ordered sample observations).

1.2 In situations other than those enumerated in **1.1**, it may be more appropriate to conduct test for non-normality of observations which are to be covered in a separate standard.

2. TERMINOLOGY

2.0 For the purpose of this standard, the following definitions shall apply.

2.1 Sample — Collection of items from a lot.

2.2 Sample Size (n) — Number of items in a sample.

2.3 Mean (Arithmetic) ($\bar{x}$) — Sum of the values of the observations divided by the number of observations.

2.4 Standard Deviation (s) — The square root of the quotient obtained by dividing sum of the squares of deviations of the observations from their mean by one less than the number of observations in the sample.

2.5 Variance — Square of the standard deviation.

*Rules for rounding off numerical values (*revised*).

2.6 Range — The difference between the largest and the smallest observations in a sample.

2.7 Null Hypothesis — Hypothesis (or assumption) that all the observations in the sample come from the same parent population, distribution or lot.

2.8 Level of Significance — The probability (or risk) of rejecting the null hypothesis when it is true. Conventionally it is taken to be 5 or 1 percent.

2.9 Statistic — A function calculated from the observations in the sample.

2.10 Critical Value — The value of the appropriate statistic which would be exceeded by chance with some small probability equivalent to the level of significance chosen.

2.11 Degrees of Freedom — The number of independent component values which are necessary to determine a statistic.

3. TESTS FOR SINGLE OUTLIER

3.0 There are many situations when, in a given sample of size n, one of the observations, which is either the largest or the smallest, is suspect. To detect such outliers, two tests have been given. The first one needs the calculation of the average and the standard deviation of the sample observations whereas the second one is based on order statistics and hence is easier to operate.

3.1 Let x_1, $x_2 \ldots\ldots x_n$ be the n sample observations arranged in the ascending order of magnitude so that $x_1 \leqslant x_2 \leqslant \ldots\ldots\ldots \leqslant x_n$. If the smallest observation x_1 is suspect, the test criterion $T_1 = \frac{\bar{x} - x_1}{s}$ may be used, where the mean ($\bar{x}$) and the standard deviation (s) of the observations are given as:

$$\bar{x} = \frac{x_1 + x_2 \ldots\ldots\ldots x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$$

If the calculated value of T_1 is greater than or equal to the corresponding tabulated (critical) value of T_1 in Table 1 for sample size n and the chosen level of significance, x_1 shall be treated as an outlier and rejected; otherwise no doubt would be cast on the validity of x_1.

3.1.1 If the largest observation x_n is doubtful, then the criterion $T_n = \frac{x_n - \bar{x}}{s}$ shall be calculated. If the calculated value of T_n is greater than or equal to the corresponding critical value in Table 1 for sample size n and the specified level of significance, x_n would be rejected as an outlier; not otherwise.

3.1.2 *Example 1* — The tensile strength (in megapascals, MPa) of 10 specimens of brass rods of 10 mm diameter for general engineering purposes obtained from a lot were obtained as 368, 370, 370, 370, 372, 372, 372, 380, 384 and 397. The doubtful observation is the highest value x_{10}=397 MPa. The mean ($\bar{x}$) and the standard deviation (s) for the 10 results are obtained as:

$$\bar{x} = 375{\cdot}5 \text{ MPa, and}$$
$$s = 9{\cdot}06 \text{ MPa.}$$

$$\text{Then } T_{10} = \frac{397 - 375{\cdot}5}{9{\cdot}06} = \frac{21{\cdot}5}{9{\cdot}06} = 2{\cdot}373$$

Since the calculated value (2·373) of T_{10} is greater than the tabulated value 2·176 of T_{10} in Table 1 for the sample size 10 and 5 percent level of significance, the largest value 397 may be treated as an outlier and rejected.

3.2 The second criterion for the detection of a single outlier is based on the ratios of differences between relevant ordered observations. If x_1, x_2......x_n are the ordered set of observations (arranged in the ascending order of magnitude) then depending upon the total number of observations in the sample, certain ratios of differences shall be calculated as indicated in Table 2. If these ratios exceed the corresponding critical values at the chosen level of significance, the existence of an outlier is indicated; not otherwise.

The ratio to be calculated is indicated as r_{10} when the sample size is 7 or less. For sample sizes from 8 to 10, the ratio to be calculated is r_{11}. For sample sizes from 11 to 13, the relevant ratio for computation is r_{21} whereas for samples from 14 to 25, the ratio r_{22} is to be computed. (The connotation of r's when the smallest value or the largest value is suspect is indicated in Table 2.)

3.2.1 *Example 2* — If the second criterion (*see* **3.2**) is applied to the data of Example 1 to find out whether the largest observation is an outlier or not, then the ratio to be used for a sample of size 10 is:

$$r_{11} = \frac{x_n - x_{n-1}}{x_n - x_2} = \frac{397 - 384}{397 - 370} = \frac{13}{27} = 0{\cdot}481$$

Since the calculated value (0·481) of r_{11} is greater than the tabulated value 0·477 in Table 2 for sample size 10 and 5 percent level of significance, the largest value 397 may be treated as an outlier. This inference is the same as drawn on the basis of the first criterion in Example 1.

4. TEST FOR TWO OR MORE OUTLIERS (AT EITHER END)

4.0 When two or more observations at the lower end or the upper end of the ordered set of results deviate unduly from the rest, one way of detecting the outliers is by the repeated application of the tests recommended in **3** for single outlier. Although this procedure is simple, it has the disadvantage that the overall level of significance of the test would not be the same as in the case of test for a single outlier. Hence this test should be applied with caution. However, separate tests have also been developed when more than one observation at either end of the ordered set of observations are suspected to be outliers.

4.1 Let x_1, x_2.........x_n be the n observations in the sample arranged according to the ascending order of magnitude. If it is desired to test whether the k largest observations are outliers, the procedure as given below may be followed:

The mean ($\bar{x}$) and the sum of the squares (S^2) of the deviations of the observations from their mean are calculated as follows:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$S^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 = (n-1) s^2$$

Leaving out the k suspect observations, the mean ($\bar{x}_{n-k}$) and the sum of squares (S^2_{n-k}) of the remaining ($n-k$) observations are calculated as:

$$\bar{x}_{n-k} = \frac{x_1 + x_2 + \ldots\ldots\ldots x_{n-k}}{n-k} = \sum_{i=1}^{n-k} x_i$$

$$S^2_{n-k} = \sum_{i=1}^{n-k} (x_i - \bar{x}_{n-k})^2$$

The ratio $L_k = \frac{S^2_{n-k}}{S^2}$, which is the test criterion, is then computed. The calculated value of L_k is compared with the corresponding critical value given in Table 3 for the sample size n and the chosen level of significance. If the calculated value of L_k is found to be less than the corresponding tabulated value, then the k largest observations will be considered as outliers, not otherwise.

It may be noted that for the criterion L_k and the Table 3, the smaller values denote the significance and not the larger values as in the case of the earlier criteria and Tables 1 and 2.

4.1.1 In case the k smallest observations happen to be suspect, the procedure followed would be exactly similar to that given in **4.1**.

4.1.2 *Example 3* — In the course of preparation of standard reference sample of bauxite, 13 splits were analyzed by a laboratory for the percentage content of silicon dioxide (SiO_2). The test results were 3·74, 3·76, 3·78, 3·78, 3·78, 3·84, 3·84, 3·85, 3·89, 3·90, 3·90, 3·98 and 4·01.

The two largest observations, namely, 3·98 and 4·01 are suspected to be outliers.

From all the 13 observations, $\bar{x}_{13} = 3{\cdot}85$ and $S^2_{13} = 0{\cdot}084$

Leaving out the 2 suspect results from the remaining 11 observations,

$$\bar{x}_{11} = 3{\cdot}82 \text{ and } S^2_{11} = 0{\cdot}034$$

$$\text{Hence } L_2 = \frac{S^2_{11}}{S^2_{13}} = \frac{0{\cdot}034}{0{\cdot}084} = 0{\cdot}405$$

Since the calculated value (0·405) of L_2 is greater than the tabulated value 0·337 of L_2 in Table 3 for the sample size 13 and 5 percent level of significance, it is inferred that there is not enough evidence to suspect the two largest observations as outliers.

5. TESTS FOR TWO OR MORE OUTLIERS (AT LEAST ONE OUTLIER AT EACH END)

5.0 When there are two or more suspect observations such that they occur at both ends of the ordered set of sample results, two statistical tests of significance for their detection have been given. The first test which is based on the ratio of the sample range to the sample standard deviation is applicable only when two outliers are present (one each at the two ends of the ordered sample results). The second test of significance is more broad-based and is applicable to all situations where two or more suspect observations occur in such a manner that at each end of the ordered sample results there is at least one suspect observation.

5.1 Test for Two Outliers (One Outlier at Each of the Two Ends) — Let $x_1, x_2, \ldots\ldots x_n$ be the n sample observations arranged in the ascending order of magnitude. When both x_1 and x_n are the suspect observations, the ($\bar{x}$), the standard deviation (s) and the range (R) are first calculated, the range being given by $R = x_n - x_1$.

The ratio R/s, which constitutes the test criterion, is then calculated and compared against the critical values given in Table 4 for the sample size n and the specified level of significance. If the calculated value of R/s is larger than the corresponding tabulated value, then both x_1 and x_n are considered to be outliers, not otherwise.

5.1.1 *Example 4* — The shearing strength (in kg) of 15 specimens obtained from a batch of plywood tea-chest panels when arranged according to ascending order of magnitude were as follows:

87·5, 88·7, 92·9, 93·3, 93·6, 94·5, 94·7, 95·0, 95·2, 95·4, 96·1, 97·2, 98·3, 100·0, 105·7

If the smallest and the largest observations, namely, 87·5 and 105·7, are suspected to be outliers, then from all the 15 observations range (R) and standard deviation (s) are obtained as:

$$R = 105{\cdot}7 - 87{\cdot}5 = 18{\cdot}2$$

$$s = \sqrt{\frac{\sum_{1}^{15} (x_i - \bar{x})^2}{14}} = 4{\cdot}32$$

$$\text{Hence } R/s = \frac{18{\cdot}2}{4{\cdot}32} = 4{\cdot}21$$

Since the tabulated value of R/s in Table 4 corresponding to a sample size 15 and 5 percent level of significance is 4·17 (which is less than the calculated value of 4·21), the smallest and largest values are considered as outliers.

5.2 Test for More Than Two Outliers (At Least One Outlier at Each End) — Let x_1, x_2 ...... x_n be the sample observations arranged in the ascending order of magnitude. If two or more (say k) of these observations are suspect such that there is at least one suspect observation at each end of the ordered observations in the sample, the procedure as given below may be followed:

The mean ($\bar{x}$) of the sample observations is first calculated. Then the residuals, that is, the absolute deviations of the observations from their mean are calculated as:

$$|x_1 - \bar{x}|, \quad |x_2 - \bar{x}|, \quad |x_3 - \bar{x}| \quad \ldots\ldots |x_n - \bar{x}|$$

These n residuals are then arranged in the ascending order of magnitude and designated as z_1, z_2 .... z_n. Thus z_1 corresponds to that sample observation x which is nearest to the mean $\bar{x}$ and z_n to the sample observation x which is farthest from the mean $\bar{x}$. The mean $\bar{z}$ and the sum of squares (U^2) of these z values are then calculated as follows:

$$\bar{z} = \frac{\sum_{i=1}^{n} z_i}{n}$$

$$U^2 = \sum_{i=1}^{n} (z_i - \bar{z})^2$$

Leaving out these z values (k in number) which correspond to the k suspect observations of the original sample, the remaining ($n - k$) values are taken and their mean ($\bar{z}_{n-k}$), that is, the mean of the ($n - k$) least extreme values, as also the sum of squares (U^2_{n-k}) are calculated as:

$$\bar{z}_{n-k} = \frac{\sum_{i=1}^{n-k} z_i}{n-k}$$

$$U^2_{n-k} = \sum_{i=1}^{n-k} (z_i - \bar{z}_{n-k})^2$$

The ratio $E_k = \frac{U^2_{n-k}}{U^2}$, which is the test criterion, is then compared with the critical value given in Table 5 for the sample size n, the value of k and the desired level of significance. If the calculated value of E_k is found to be smaller than the corresponding tabulated value, then the k original suspect values are considered to be outliers, not otherwise.

It may be noted that smaller values of E_k denote significance as in the case of L_k in Table 3.

5.2.1 *Example 5* — In the data on shearing strength given in Example 4, if the smallest two observations 87·5 and 88·7 and the largest observation 105·7 are suspect to be outliers, then the mean ($\bar{x}$) is first calculated as 95·2.

The residuals, that is, the absolute deviations of the sample observations from their mean are then found out as 7·7, 6·5, 2·3, 1·9, 1·6, 0·7, 0·5, 0·2, 0, 0·2, 0·9, 2·0, 3·1, 4·8 and 10·5.

If these residuals (z_i) are arranged in the ascending order of magnitude, we get 0, 0·2, 0·2, 0·5, 0·7, 0·9, 1·6, 1·9, 2·0, 2·3, 3·1, 4·8, 6·5, 7·7 and 10·5.

Now the 3 residuals corresponding to the 3 original suspected outliers in the new sample are the last three values, namely, 6·5, 7·7 and 10·5 which would be suspect and tested for their significance.

From all the 15 residuals we get the mean ($\bar{z}_{15}$) as

$$\bar{z}_{15} = 2{\cdot}86$$

$$U^2_{15} = \sum_{i=1}^{15} (z_i - \bar{z}_{15})^2 = 138{\cdot}84$$

Leaving out the 3 largest residuals (z values), we get from the remaining 12 values:

$$\bar{z}_{12} = 1{\cdot}52$$

$$U^2_{12} = \sum_{i=1}^{12} (z_i - \bar{z}_{12})^2 = 22{\cdot}14$$

Hence $E_3 = \frac{U^2_{12}}{U^2_{15}} = \frac{22{\cdot}14}{138{\cdot}84} = 0{\cdot}159$

Since the calculated value (0·159) of E_3 is less than 0·206 which is the tabulated value of E_k in Table 5 for the sample size 15, $k = 3$ and 5 percent level of significance, all the three observations, namely, 87·5, 88·7 and 105·7, are to be considered as outliers.

TABLE 1 CRITICAL VALUES OF T_1 OR T_n FOR TESTING AN OUTLIER

(*Clause* 3.1)

Sample Size, n	Significance Level	
	5 Percent	1 Percent
3	1·153	1·155
4	1·463	1·492
5	1·672	1·749
6	1·822	1·944
7	1·938	2·097
8	2·032	2·221
9	2·110	2·323
10	2·176	2·410
11	2·234	2·485
12	2·285	2·550
13	2·331	2·607
14	2·371	2·659
15	2·409	2·705
16	2·443	2·747
17	2·475	2·785
18	2·504	2·821
19	2·532	2·854
20	2·557	2·884
21	2·580	2·912
22	2·603	2·939
23	2·624	2·963
24	2·644	2·987
25	2·663	3·009
30	2·745	3·103
35	2·811	3·178
40	2·866	3·240
45	2·914	3·292
50	2·956	3·336

TABLE 2 CRITICAL VALUES OF CRITERIA BASED ON ORDER STATISTICS FOR TESTING AN OUTLIER

(*Clause* 3.2)

Ratio	When the Outlier is		Sample Size, n	Significance Level	
	Smallest Value	Largest Value		5 Percent	1 Percent
r_{10}	$\frac{x_2 - x_1}{x_n - x_1}$	$\frac{x_n - x_{n-1}}{x_n - x_1}$	3	0·941	0·988
			4	0·765	0·889
			5	0·642	0·780
			6	0·560	0·698
			7	0·507	0·637
r_{11}	$\frac{x_2 - x_1}{x_{n-1} - x_1}$	$\frac{x_n - x_{n-1}}{x_n - x_2}$	8	0·554	0·683
			9	0·512	0·635
			10	0·477	0·597
r_{21}	$\frac{x_3 - x_1}{x_{n-1} - x_1}$	$\frac{x_n - x_{n-2}}{x_n - x_2}$	11	0·576	0·679
			12	0·546	0·642
			13	0·521	0·615
r_{22}	$\frac{x_3 - x_1}{x_{n-2} - x_1}$	$\frac{x_n - x_{n-2}}{x_n - x_3}$	14	0·546	0·641
			15	0·525	0·616
			16	0·507	0·595
			17	0·490	0·577
			18	0·475	0·561
			19	0·462	0·547
			20	0·450	0·535
			21	0·440	0·524
			22	0·430	0·514
			23	0·421	0·505
			24	0·413	0·497
			25	0·406	0·489

TABLE 3 CRITICAL VALUES OF L_k FOR THE DETECTION OF k (LARGEST OR SMALLEST) OUTLIERS

(*Clause* 4.1)

k	2		3		4		5		6		7		8		9		10	
Significance Level / Sample Size, n	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
4	0·001	0·000																
5	0·018	0·004																
6	0·055	0·021	0·010	0·002														
7	0·106	0·047	0·032	0·010														
8	0·146	0·076	0·064	0·028	0·022	0·008												
9	0·194	0·112	0·099	0·048	0·045	0·018												
10	0·233	0·142	0·129	0·070	0·070	0·032	0·034	0·012										
11	0·270	0·178	0·162	0·098	0·098	0·052	0·054	0·026										
12	0·305	0·208	0·196	0·120	0·125	0·070	0·076	0·038	0·042	0·019								
13	0·337	0·233	0·224	0·147	0·150	0·094	0·098	0·056	0·060	0·033								
14	0·363	0·267	0·250	0·172	0·174	0·113	0·122	0·072	0·079	0·042	0·050	0·027						
15	0·387	0·294	0·276	0·194	0·197	0·132	0·140	0·090	0·097	0·057	0·066	0·037						
16	0·410	0·311	0·300	0·219	0·219	0·151	0·159	0·108	0·115	0·072	0·082	0·049	0·055	0·030				
17	0·427	0·338	0·322	0·237	0·240	0·171	0·181	0·126	0·136	0·091	0·100	0·064	0·072	0·044				
18	0·447	0·358	0·337	0·260	0·259	0·192	0·200	0·140	0·154	0·104	0·116	0·076	0·086	0·053	0·062	0·056		
19	0·462	0·366	0·354	0·272	0·277	0·201	0·209	0·154	0·168	0·118	0·130	0·088	0·099	0·064	0·074	0·046		
20	0·484	0·387	0·377	0·300	0·299	0·231	0·238	0·175	0·188	0·136	0·150	0·104	0·115	0·078	0·088	0·058	0·066	0·042
25	0·550	0·488	0·450	0·377	0·374	0·308	0·312	0·246	0·262	0·204	0·222	0·168	0·184	0·144	0·154	0·112	0·126	0·092
30	0·599	0·526	0·506	0·434	0·434	0·369	0·376	0·312	0·327	0·368	0·283	0·229	0·245	0·196	0·212	0·166	0·183	0·142
35	0·642	0·574	0·554	0·484	0·482	0·418	0·424	0·364	0·376	0·321	0·334	0·282	0·297	0·250	0·264	0·220	0·235	0·194
40	06·72	0·608	0·588	0·522	0·523	0·460	0·468	0·408	0·421	0·364	0·378	0·324	0·342	0·292	0·310	0·262	0·280	0·234
45	0·696	0·636	0·618	0·558	0·556	0·498	0·502	0·444	0·456	0·399	0·417	0·361	0·382	0·328	0·350	0·296	0·320	0·270
50	0·722	0·668	0·646	0·592	0·588	0·531	0·535	0·483	0·490	0·438	0·450	0·400	0·414	0·368	0·383	0·336	0·356	0·368

TABLE 4 CRITICAL VALUE OF R/s FOR THE DETECTION OF TWO OUTLIER (ONE AT EACH END OF ORDERED RESULTS)

(*Clauses* 5.1 *and* 5.1.1)

SAMPLE SIZE, n	SIGNIFICANCE LEVELS	
	5 Percent	1 Percent
3	2·00	2·00
4	2·43	2·45
5	2·75	2·80
6	3·01	3·10
7	3·22	3·34
8	3·40	3·54
9	3·55	3·72
10	3·68	3·88
11	3·80	4·01
12	3·91	4·13
13	4·00	4·24
14	4·09	4·34
15	4·17	4·43
16	4·24	4·51
17	4·31	4·59
18	4·38	4·66
19	4·43	4·73
20	4·49	4·79
30	4·89	5·25
40	5·15	5·54
50	5·35	5·77

TABLE 5 CRITICAL VALUES OF E_k FOR THE DETECTION OF k (SOME SMALL AND OTHERS LARGE) OUTLIERS

(*Clauses* 5.2 *and* 5.2.1)

k	2		3		4		5		6		7		8		9		10	
Significance Level / Sample Size, n	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
4	0·001	0·000																
5	0·010	0·002																
6	0·034	0·012	0·004	0·001														
7	0·065	0·028	0·016	0·006														
8	0·099	0·050	0·034	0·014	0·010	0·004												
9	0·137	0·078	0·057	0·026	0·021	0·009												
10	0·172	0·101	0·083	0·037	0·037	0·013	0·014											
11	0·204	0·134	0·107	0·064	0·055	0·030	0·026	0·012										
12	0·234	0·159	0·133	0·083	0·073	0·042	0·039	0·020	0·018	0·008								
13	0·262	0·181	0·156	0·103	0·092	0·056	0·053	0·031	0·028	0·014								
14	0·293	0·207	0·179	0·123	0·112	0·072	0·068	0·042	0·039	0·022	0·021	0·012						
15	0·317	0·238	0·206	0·146	0·134	0·090	0·084	0·054	0·052	0·032	0·030	0·018						
16	0·340	0·263	0·227	0·166	0·153	0·107	0·102	0·068	0·067	0·040	0·041	0·024	0·024	0·014				
17	0·362	0·290	0·248	0·188	0·170	0·122	0·116	0·079	0·078	0·052	0·050	0·032	0·032	0·018				
18	0·382	0·306	0·267	0·206	0·187	0·141	0·132	0·094	0·091	0·062	0·062	0·041	0·041	0·026	0·026	0·014		
19	0·398	0·323	0·287	0·219	0·203	0·156	0·146	0·108	0·105	0·074	0·074	0·050	0·050	0·032	0·033	0·020		
20	0·416	0·339	0·302	0·236	0·221	0·170	0·163	0·121	0·119	0·086	0·085	0·058	0·059	0·040	0·041	0·026	0·028	0·017
25	0·493	0·418	0·381	0·320	0·298	0·245	0·236	0·188	0·186	0·146	0·146	0·110	0·114	0·087	0·089	0·066	0·068	0·050
30	0·549	0·482	0·443	0·386	0·364	0·308	0·298	0·250	0·246	0·204	0·203	0·166	0·166	0·132	0·137	0·108	0·112	0·087
35	0·596	0·533	0·495	0·435	0·417	0·364	0·351	0·299	0·298	0·252	0·254	0·211	0·214	0·132	0·181	0·149	0·154	0·124
40	0·629	0·574	0·534	0·480	0·458	0·408	0·395	0·347	0·343	0·298	0·297	0·258	0·259	0·177	0·223	0·190	0·195	0·164
45	0·658	0·607	0·567	0·518	0·492	0·446	0·433	0·386	0·381	0·336	0·337	0·294	0·299	0·220	0·263	0·228	0·233	0·200
50	0·684	0·636	0·599	0·550	0·529	0·482	0·468	0·424	0·417	0·376	0·373	0·334	0·334	0·257	0·299	0·264	0·268	0·235