Annexure 12.5: Method of Network Analysis

(a) Hardy Cross Method

(i) Balancing Heads

In this method, from the knowledge of system inflows and outflows, the flows in all the pipes of the network are distributed to meet continuity constraints at all the nodes. When inflows and outflows are explicitly known, this will involve initial assigning of flows in one of the pipes of every primary loop in the system. Based on this assignment, flows in other pipes of the loops are assigned. The requirement that the sum of head losses around all primary loops should equal zero gives rise to a system of as many equations as number of loops. The requirement of total head loss between source nodes is satisfied by considering additional pseudo-pipe (an imaginary infinite resistance pipe connecting the two source nodes to form a pseudo-loops) Solution of the exactly determined system of non-linear equations is affected by a systematic relaxation in the Hardy Cross method. In the Hardy Cross method of balancing heads, which is a trial-and-error process, the correction factor for assumed flows (necessary formulae are made algebraically consistent by arbitrarily assigning positive signs to clockwise flows and associated head losses and negative signs to anti-clockwise flows and associated head losses) Q in a circuit is calculated by the formula:

$$\Delta Q = \frac{-\sum h}{n.\sum h/_{Q}} \qquad \text{(Eq. 12.1a)}$$

where ΔQ= is loop flow correction.

The assumed flows are corrected accordingly, and the procedure is repeated until the required degree of precision is reached. This is essentially a repetitive procedure. The sequential steps are presented below:

a) Assume suitable values of flow Q in each pipeline such that the flows coming into each junction of the loop are equal to flows leaving the junction.

b) Assign positive sign to all clockwise flows and negative sign to all anti-clockwise flow.

c) Compute the head loss H in each pipe by use of the friction formula.

d) Compute ∑h (i.e., algebraic sum of the head losses) around each loop and if this is nearly equal to zero in all loops (within allowable limits of ±0.01 m), the assumed flows are correct.

e) Otherwise, if ∑h is not equal to 0 for any loop, compute the error in loop flow using Eq. 12.1a, for real as well as pseudo-loops.

f) Pipes operating in more than one circuit draw corrections from each circuit. However, the second correction is of the opposite sign as applied to the first circuit.

g) Repeat the cycle, till ∑h (around each loop) is nearly equal to zero within the allowable limits. Then, the final values of flows are the actual values in the pipelines.

(ii) Balancing Flows

When using the method of balancing flows at junctions or nodes of the system, pressures at nodes are assumed on the basis of given pressure surface elevations at some nodes (e.g., fixed elevation reservoirs) and the flows in the pipes are estimated.

In the method of balancing flows (modification of original Hardy Cross Method), which is applicable to junctions and nodes, the flows at each junction are made to balance for the assumed heads at the junctions and the corresponding head losses in the pipes. The correction factor for assumed head losses in the pipes is calculated using the formula:

$$\Delta H = \frac{\sum Q}{\sum Q/_{n\,h}} \qquad \text{Eq. (12.2a)}$$

The steps in the computation are as under:

(i) Assume heads at all the free junctions.

(ii) Assign positive sign to head losses for flows towards the junction and negative sign to those away from the junction.

(iii) Compute the flows in each pipe by use of the friction formula.

(iv) Compute ∑Q (i.e., algebraic sum of the flows) at each fret junction and if this is nearly equal to zero at all junctions (within allowable limits of ±2%), the assumed head losses are correct.

(v) Otherwise, if ∑Q is not equal to zero at any junction, compute the correction in head by using the Eq. (12.2a).

(vi) Add the correction factor to the assumed heads with due regard to the sign of corrections.

(vii) Pipes common to more than one node receive corrections from each node.

(viii) Repeat the cycle till ∑Q= 0 at each node or junction when the final corrected values of H are obtained.

The Hardy Cross method considers one equation at a time to obtain corrections to the assumed link flows (method of balancing heads) or assumed nodal heads (method of balancing flows). The method is good for hand calculations as only one equation is considered at a time for solving. However, it is slow converging and may sometimes diverge.

(b) Newton-Raphson Method

Network balancing using Newton-Raphson method is again an iterative process, but the method seems to be faster and convergence much more rapid from a reasonably good start. The principle of this method is explained most simply by reference to solution of a single equation f(p) = 0. According to Newton's rule, if p is an approximation to a root of f(p), then (p + ∆p) is a better approximation where:

$$\Delta p \ = \frac{f(p)}{f\prime(p)} \qquad \text{Eq. (12.3a)}$$

The nature of this result can be recognised from the Taylor series expansion of f(p+∆p), viz.

$$f(p + \Delta p) = f(p) + \Delta p . f''(p) + + \cdots + \text{ terms involving higher powers of } (\Delta p) \qquad \text{Eq. (12.4a)}$$

f(p + ∆p) is equal to zero, if (p + ∆p) is in reality a solution to f(p)= 0. If, in the above equation, the terms involving powers of ∆p higher than the first are neglected, one obtains Newton's rule. The method can be extended to the solution of n simultaneous equations with n variables.

In setting up a water distribution network for balancing heads by Newton-Raphson method on the computer, it is useful to note the following steps and observations. Flows in the pipes are assumed to meet all the continuity constraints. The flows in all pipes of loop i are assumed to be in error by ΔQ_i; correction from both loops, the one coming from the loop under consideration being algebraically added, the other being algebraically deducted.

Equations to balance head losses around loops are then framed in terms of corrected flows. These equations are solved simultaneously to obtain loop flow corrections for all loops (both real and pseudo). The computed ∆Qi, are applied to all pipes of the network as explained under Hardy Cross method giving due consideration to common pipes between loops and the iteration proceeds. The programme terminates at the allowable head tolerance or when iterations exceed a certain prescribed limit.

A general computer programme for network head balance according to Newton-Raphson method is required to compute from input values and set up the coefficient matrix for solution for ΔQ_i's. The set of linear simultaneous equations could be solved by calling appropriate library subroutines. The success of the Newton-Raphson technique lies in the selection of a good starting approximation. If the approximation is poor, it can result in the divergence of the solution. Computer programmes are readily available for the Newton-Raphson technique. The method can be applied to balance flows by assuming nodal heads also as in Hardy Cross Method.

(c) Linear Theory Method

This method, proposed by Wood and Charles, is useful for network balancing through "balancing heads by correcting assumed flows". This is also an iterative method, said to converge faster than the Hardy Cross method.

In the methods of balancing described earlier, it is necessary to assume certain values for the variables to start the iterative procedure. Naturally, therefore, the number of iterations depend upon the initial guess. No such initialisation is needed in the linear theory method.

The linear theory transforms the loop head loss non-linear relationships into linear relationships by approximating the head loss in each pipe by

$$h_x = (RQ^n)_x = (RQ^{n-1})_P Q_x = (R'Q)_x \qquad \text{Eq. (12.5a)}$$

in which Q_x is the assumed flow in pipe x. Thus, the pipe resistance constant R_x is replaced by $(R')_x$ so that, $(R')_x = (RQ^{n-1})$

All the non-linear loop head loss relationships become linear. These linear equations and the node flow continuity linear equations are solved simultaneously to obtain all Q_x values. The solution, however, will not be correct as the obtained Q_x values will not be the same as assumed Q_x values. However, it is claimed that by repeating the process several times, the obtained and the assumed values will be found to be identical, thus giving the correct solution.

In the linear theory, for the first iteration, all the Q_x values are taken as 1 giving R′ = R. It is observed that in this method, if used just as suggested earlier, yields pipe flows which tend to oscillate about the final solution. To obviate this, Wood and Charles have suggested that after two iterative solutions, for all the iterations thereafter, the initial flow rates to be used in the computations should be the average of the flow rates obtained from the past two iterations. Better would be to take the average of assumed and obtained values of previous iteration (Bhave et al, 2006). Thus, for the i^{th} iteration,

$$ {}_{i+1}Q_{xa} = \frac{{}_{i}Q_{xa} + {}_{i}Qxo}{2} \qquad \text{Eq. (12.6a)} $$

in which the subscript i, i+1 denotes two successive iterations. Q_{xa} and Q_{xo} are the assumed and obtained values of Q in ant iteration.

The Newton-Raphson and Linear Theory methods, linearises the non-linear equations. While Newton-Raphson method uses truncated Taylor's series to obtain corrections to assumed pipe discharges or nodal heads successively till convergence, the Linear Theory method merges the non-linear part with resistance to linearise them and upgrade the assumed pipe discharges or nodal heads till they stabilise. Convergence in both the methods is fast. Linear theory method does not require initialisation and the iterative procedure can be started with the same values of pipe discharges in each link.

(d) Gradient Method

The gradient method solves Q-H equations by simultaneously solving set of Eqs. (12.1) and (12.2). The upgraded values of ${}_{t+1}Q$ and ${}_{t+1}H$ for any t+1 iteration can be obtained from the known values of ${}_{t}Q_{ox}$, ${}_{t}H_{oi}$ and ${}_{t}H_{oj}$ in t^{th} iteration as described below. Let ${}_{t}\Delta Hi$, ${}_{t}\Delta Hj$, and ${}_{t}\Delta Qx$ be the unknown corrections for the t^{th} iteration. The pipe head loss equation with truncated Taylor's series expansion of non-linear term can be written as

$$ \left({}_{t}H_{oi} + {}_{t}\Delta H_i\right) - \left({}_{t}H_{oj} + {}_{t}\Delta H_j\right) = R_{ox}\ {}_{t}Q_{ox}^{n} + nR_{ox}\left|{}_{t}Q_{ox}\right|^{n-1}{}_{t}\Delta Q_x, x = 1,\ldots,X \qquad \text{(Eq. 12.7a)} $$

in which R_{ox} = known resistance constant of pipe x.

Rewriting Eq. (12.11) by transferring fixed nodal head terms, if any, on the right-hand side and term containing ΔQ_x on the left-hand side

$$ {}_{t+1}H_i - {}_{t+1}H_j - nR_{ox}\left|{}_{t}Q_{ox}\right|^{n-1}{}_{t}\Delta Q_x = R_{ox}\ {}_{t}Q_{ox}^{n}, x = 1,\ldots,X \qquad \text{(Eq. 12.8a)} $$

Subtracting $nR_{ox}\ {}_{t}Q_{ox}^{n}$ from either side

$$ {}_{t+1}H_i - {}_{t+1}H_j - nR_{ox}\left|{}_{t}Q_{ox}\right|^{n-1}\left({}_{t}Q_{ox} + {}_{t}\Delta Q_x\right) = (1-n)R_{ox}\ {}_{t}Q_{ox}^{n}, x = 1,\ldots,X \qquad \text{(Eq. 12.9a)} $$

Replacing ${}_{t}Q_{ox}$ + ${}_{t}\Delta Q_x$ by ${}_{t+1}Q_x$

$$ {}_{t+1}H_i - {}_{t+1}H_j - \left(nR_{ox}\left|{}_{t}Q_{ox}\right|^{n-1}\right){}_{t+1}Q_x = (1-n)R_{ox}\ {}_{t}Q_{ox}^{n}, x = 1,\ldots,X \qquad \text{(Eq. 12.10a)} $$

Equation (12.10a) provides X number of linearised equations involving corrected values of pipe discharges and nodal heads as unknowns.

Linear node flow continuity equations can be written for corrected discharge values as

$$\sum_{x \text{ connected to } j} {}_{t+1}Q_x + q_{oj} = 0, j = M+1,...,M+N \quad \text{(Eq.12.11a)}$$

which are *N* linear equations. Solution of set of Eqs. (12.14) and (12.15) provides the corrected values of *X* pipe discharges and *N* unknown nodal heads.

Pipe discharge ${}_tQ_{ox}$ can be taken as unity for the first iteration or can be alternatively taken as some other arbitrarily chosen value.

Todini and Pilati (1987) suggested matrix form of Gradient method and showed that by starting with unit flow in all pipes, improved values of nodal heads and nodal flows can be iteratively obtained by solving the following equations in the matrix form. The iterative procedure can be terminated when no or negligible change in the values in two successive iterations are obtained.

The improved values of nodal heads **H** in matrix form are given by:

$${}_{t+1}\mathbf{H} = -[\mathbf{A}_{21}(\mathbf{N}\mathbf{A}_{11})^{-1}\mathbf{A}_{12}]^{-1}[\mathbf{A}_{21}(\mathbf{N}\mathbf{A}_{11})^{-1}(\mathbf{A}_{11}\,{}_t\mathbf{Q} + \mathbf{A}_{10}\mathbf{H}_0) - (\mathbf{A}_{21}\,{}_t\mathbf{Q} - \mathbf{q}_0)] \quad \text{Eq. (12.12a)}$$

$${}_{t+1}\mathbf{Q} = (\mathbf{I} - \mathbf{N}^{-1})\,{}_t\mathbf{Q} - [\mathbf{N}^{-1}\mathbf{A}_{11}^{-1}(\mathbf{A}_{12}\,{}_{t+1}\mathbf{H} + \mathbf{A}_{10}\mathbf{H}_0)] \quad \text{Eq. (12.13a)}$$

In which **N** and $\mathbf{A}_{11}$ are diagonal matrix of size (X, X); $\mathbf{A}_{12} = \mathbf{A}_{21}^T$ is unknown-head node incidence matrix of size (X, N); $\mathbf{A}_{22}$ = 0, a small null matrix of size (N, N); **Q** is column matrix of unknown pipe discharges of size (X, 1); **H** is column matrix of unknown nodal heads of size (N, 1); $\mathbf{A}_{10}$ is known head node incidence matrix of size (X, M); $\mathbf{H}_0$ is column matrix of known nodal heads of size (M, 1); **Q** is column matrix of known or assumed pipe discharges of size (X, 1); q_0 is column matrix of known nodal demands of size (N, 1); I is an identity matrix.

The gradient method is basically an application of Newton-Raphson method to simultaneously obtain unknown Q and H. The gradient method is also fast converging and like Linear Theory method, it does not require initialisation. The hydraulic solver EPANET is based on Gradient Method (Rossman 2001). Formulation and solution using matrix method is explained with a simple example. Consider a two-source network as shown in Figure 1. The network has three demand nodes - 3, 4, and 5. Total number of pipes are six.

The HGL at the source nodes 1 and 2 are: H_1 =100 m, and H_2 = 95 m. The length of pipes 1 to 6 are: L_1= 200 m, L_2 = 200 m, L_3=200 m, L_4 = 200 m, L_5 = 300 m, and L_6 = 300 m. The pipe diameters for pipes 1 to 6 are: D_1 = 250 mm, D_2 = 300 mm, D_3 = 300 mm, D_4 = 250 mm, D_5=250 mm, and D_6 = 250 mm. The H-W constant for all the pipes is 100. The nodal demands at nodes 3 to 5 are q_3 = 0.3 m^3/s, q_4 = 0.2 m^3/s, and q_5 = 0.1 m^3/s.

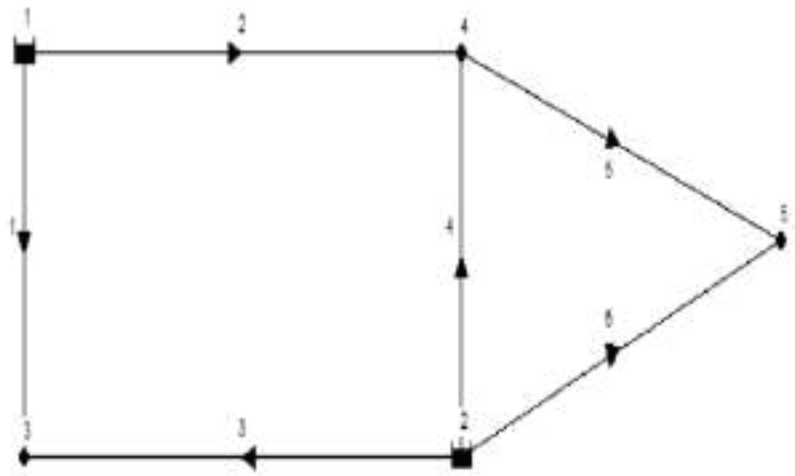

Fig. 1 Two-source, three-demand-node looped network

For the network, the number of pipes, X=6; the number of source nodes, M=2; and the number of demand nodes, N=3. Let the assumed values of pipe discharges for the first iteration be Q_1, …, Q_6 = 1.

The matrix method alternatively determines improved values of heads and flows using the following formulation:

$$_{t+1}\mathbf{H} = -\mathbf{A}_{21}\,(\mathbf{N}\mathbf{A}_{11})^{-1}\,\mathbf{A}_{12})^{-1}\,[\mathbf{A}_{21}\,(\mathbf{N}\mathbf{A}_{11})^{-1}\,(\mathbf{A}_{11}\mathbf{Q} + \mathbf{A}_{10}\,\mathbf{H}_0) - (\mathbf{A}_{21}\mathbf{Q} - \mathbf{q}_0)]$$

$$_{t+1}\mathbf{Q} = (\mathbf{I} - (\mathbf{N}\mathbf{A}_{11})^{-1}\,\mathbf{A}_{11})\,\mathbf{Q} - (\mathbf{N}\mathbf{A}_{11})^{-1}\,(\mathbf{A}_{12}\,_{t+1}\,\mathbf{H} + \mathbf{A}_{10}\,\mathbf{H}_0)$$

Matrices $\mathbf{A}_{12}$, $\mathbf{A}_{10}$, **Q** and **R** for the network are given by

$$\mathbf{A}_{12}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\1 & 0 & 0\\0 & 1 & 0\\0 & -1 & 1\\0 & 0 & 1\end{bmatrix};\quad \mathbf{A}_{10}=\begin{bmatrix}-1 & 0\\-1 & 0\\0 & -1\\0 & -1\\0 & 0\\0 & -1\end{bmatrix};\quad \mathbf{Q}=\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix};\quad \mathbf{R}=\begin{bmatrix}257.626\\106.0174\\106.0174\\257.626\\386.439\\386.439\end{bmatrix}$$

Matrices $\mathbf{A}_{12}$ and $\mathbf{A}_{10}$ shows connectivity between nodes and pipes. Rows belongs to pipes and columns belongs to nodes. In column 1 of $\mathbf{A}_{12}$, connectivity at node 3 is shown. Pipes 1 and 3 are marked as 1 and others as 0. Both 1 and 3 are marked as positive as both are supply pipes at node 3. In case of outgoing pipe, pipe is marked negative. **Q** is column matrix with initial assumed value as 1. Pipe resistances are calculated, and matric **R** is framed.

Diagonal matrix A_{11}, in which diagonal term is $R_x|Q|^{n-1}$, is:

$$\mathbf{A_{11}} = \begin{bmatrix} 257.626 & 0 & 0 & 0 & 0 & 0 \\ 0 & 106.0174 & 0 & 0 & 0 & 0 \\ 0 & 0 & 106.0174 & 0 & 0 & 0 \\ 0 & 0 & 0 & 257.626 & 0 & 0 \\ 0 & 0 & 0 & 0 & 386.439 & 0 \\ 0 & 0 & 0 & 0 & 0 & 386.439 \end{bmatrix}$$

Matrix $\mathbf{A_{21}}$, which is transpose of matrix $\mathbf{A_{12}}$, is

$$\mathbf{A_{21}} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Diagonal matrix **N**, in which the diagonal term is n is

$$\mathbf{N} = \begin{bmatrix} 1.852 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.852 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.852 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.852 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.852 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.852 \end{bmatrix}$$

Matrix product $\mathbf{NA_{11}}$ is

$$\mathbf{NA_{11}} = \begin{bmatrix} 477.1234 & 0 & 0 & 0 & 0 & 0 \\ 0 & 196.3442 & 0 & 0 & 0 & 0 \\ 0 & 0 & 196.3442 & 0 & 0 & 0 \\ 0 & 0 & 0 & 477.1234 & 0 & 0 \\ 0 & 0 & 0 & 0 & 715.6851 & 0 \\ 0 & 0 & 0 & 0 & 0 & 715.6851 \end{bmatrix}$$

Inverse of $\mathbf{(NA_{11})}$ is

$$\mathbf{(NA_{11})^{-1}} = \begin{bmatrix} 0.002096 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.005093 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.005093 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.002096 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.001397 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.001397 \end{bmatrix}$$

Matrix product $\mathbf{A_{21}\,(NA_{11})^{-1}}$ is,

$$\mathbf{A_{21}\,(NA_{11})^{-1}} = \begin{bmatrix} 0.002096 & 0 & 0.005093 & 0 & 0 & 0 \\ 0 & 0.005093 & 0 & 0.002096 & -0.0014 & 0 \\ 0 & 0 & 0 & 0 & 0.001397 & 0.001397 \end{bmatrix}$$

Matrix $\mathbf{A_{21} (NA_{11})^{-1} A_{12}}$ is,

$$\mathbf{A_{21} (NA_{11})^{-1} A_{12}} = \begin{bmatrix} 0.007189 & 0 & 0 \\ 0 & 0.008586 & -0.0014 \\ 0 & -0.0014 & 0.002795 \end{bmatrix}$$

Matrix $\mathbf{-(A_{21} (NA_{11})^{-1} A_{12})^{-1}}$ is given by,

$$\mathbf{-(A_{21} (NA_{11})^{-1} A_{12})^{-1}} = \begin{bmatrix} -139.102 & 0 & 0 \\ 0 & -126.781 & -63.3905 \\ 0 & -63.3905 & 389.538 \end{bmatrix}$$

Matrices $\mathbf{A_{11}Q}$, $\mathbf{A_{10} H_0}$ and their addition $\mathbf{A_{11}Q + A_{10} H_0}$ are given by

$$\mathbf{A_{11}Q} = \begin{bmatrix} 257.626 \\ 106.0174 \\ 106.0174 \\ 257.626 \\ 386.439 \\ 386.439 \end{bmatrix} ; \mathbf{A_{10} H_0} = \begin{bmatrix} -100 \\ -100 \\ -95 \\ -95 \\ 0 \\ -95 \end{bmatrix} ; \mathbf{A_{11}Q + A_{10} H_0} = \begin{bmatrix} 157.626 \\ 6.017381 \\ 11.01738 \\ 162.626 \\ 386.439 \\ 291.439 \end{bmatrix}$$

Matrix $\mathbf{A_{21} (NA_{11})^{-1} (A_{11}Q + A_{10} H_0)}$ is given by,

$$\mathbf{A_{21} (NA_{11})^{-1} (A_{11}Q + A_{10} H_0)} = \begin{bmatrix} 0.38648 \\ -0.16846 \\ 0.947174 \end{bmatrix}$$

Matrices $\mathbf{A_{21}Q}$ and $\mathbf{A_{21}Q - q_0}$ are given by

$$\mathbf{A_{21}Q} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} ; \mathbf{A_{21}Q - q_0} = \begin{bmatrix} 1.7 \\ 0.8 \\ 1.9 \end{bmatrix}$$

Matrix $\mathbf{[A_{21} (NA_{11})^{-1} (A_{11}Q + A_{10} H_0) - (A_{21}Q - q_0)]}$ is given by,

$$\mathbf{[A_{21} (NA_{11})^{-1} (A_{11}Q + A_{10} H_0) - (A_{21}Q - q_0)]} = \begin{bmatrix} -1.31352 \\ -0.96846 \\ 0.95283 \end{bmatrix}$$

Now, matrix ${}_{t+1}\mathbf{H}$ is obtained as

$$\mathbf{{}_{t+1}H = -A_{21}\ (NA_{11})^{-1}\ A_{12})^{-1}\ [A_{21}\ (NA_{11})^{-1}\ (A_{11}Q + A_{10}\ H_0) - (A_{21}Q - q_0)]}$$

$${}_{t+1}\mathbf{H} = \begin{bmatrix} 182.7127 \\ 183.1827 \\ 432.5532 \end{bmatrix}$$

To find ${}_{t+1}\mathbf{Q}$ following step by step analysis, the following matrices are obtained.

$$\mathbf{A_{12\ t+1}\ H} = \begin{bmatrix} 182.7127 \\ 183.1827 \\ 182.7127 \\ 183.1827 \\ 249.3705 \\ 432.5532 \end{bmatrix}; \quad \mathbf{A_{12\ t+1}\ H + A_{10}\ H_0} = \begin{bmatrix} 82.71271 \\ 83.18271 \\ 87.71271 \\ 88.18271 \\ 249.3705 \\ 337.5532 \end{bmatrix}$$

$$\mathbf{(NA_{11})^{-1}\ (A_{12\ t+1}\ H + A_{10}\ H_0)} = \begin{bmatrix} 0.173357 \\ 0.423658 \\ 0.446729 \\ 0.184822 \\ 0.348436 \\ 0.47165 \end{bmatrix}$$

$$\mathbf{(NA_{11})^{-1}\ A_{11}} = \begin{bmatrix} 0.539957 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.539957 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.539957 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.539957 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.539957 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.539957 \end{bmatrix}$$

$$\mathbf{I - (NA_{11})^{-1}\ A_{11}} = \begin{bmatrix} 0.460043 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.460043 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.460043 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.460043 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.460043 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.460043 \end{bmatrix}$$

$$\mathbf{(I - (NA_{11})^{-1}\ A_{11})\ Q} = \begin{bmatrix} 0.460043 \\ 0.460043 \\ 0.460043 \\ 0.460043 \\ 0.460043 \\ 0.460043 \end{bmatrix}$$

$$ {}_{t+1}\mathbf{Q} = (\mathbf{I} - (\mathbf{NA}_{11})^{-1}\, \mathbf{A}_{11})\, \mathbf{Q} - (\mathbf{NA}_{11})^{-1}\, (\mathbf{A}_{12}\ {}_{t+1}\mathbf{H} + \mathbf{A}_{10}\, \mathbf{H}_0) = \begin{bmatrix} 0.286686 \\ 0.036386 \\ 0.013314 \\ 0.275222 \\ 0.111607 \\ -0.01161 \end{bmatrix} $$

The iterative procedure is continued further and the final solution with desired accuracy is obtained in five iterations. Iteration details are shown in the Table 1. Flow values in the fifth iteration are same up to three places after decimal. The iterative process can be continued to have more accuracy in flow values.

Table 1: *Pipe Discharges and nodal beads after different iterations*

Parameters	1	2	3	4	5
Q_1, m^3/sec	0.286686	0.166117	0.15377	0.153596	0.153596
Q_2, m^3/sec	0.036386	0.176153	0.196408	0.204748	0.205104
Q_3, m^3/sec	0.013314	0.133883	0.14623	0.146404	0.146404
Q_4, m^3/sec	0.275222	0.106857	0.051406	0.039754	0.03917
Q_5, m^3/sec	0.111607	0.08301	0.047815	0.044503	0.044273
Q_6, m^3/sec	-0.01161	0.01699	0.052185	0.055497	0.055727
H_3, m	182.7127	94.36713	92.00391	91.98018	91.98018
H_4, m	183.1827	98.14024	94.84046	94.38711	94.36166
H_5, m	432.5532	94.64127	94.01386	93.17928	93.16015